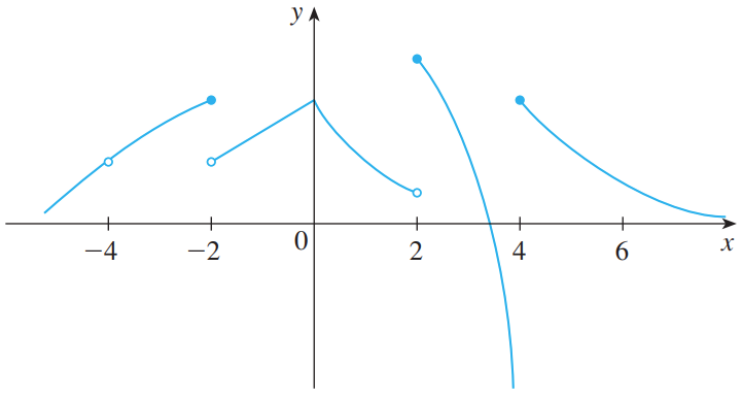
Exercise 3

(a) From the graph of f, state the numbers at which f is discontinuous and explain why.

(b) For each of the numbers stated in part (a), determine whether f is continuous from the right, or from the left, or neither.



Solution

Recall that the condition for a function to be continuous at $x = a$ is

$$\lim_{x \to a^-} f(x) = \lim_{x \to a^+} f(x) = f(a).$$

The function is discontinuous at $x = -4$ because the function is not defined there. It's neither continuous from the left nor the right.

$$\lim_{x \to -4^-} f(x) = \lim_{x \to -4^+} f(x) \neq \text{undefined}$$

The function is discontinuous at $x = -2$ because the left-hand and right-hand limits are not equal. It is continuous from the left, though.

$$f(-2) = \lim_{x \to -2^-} f(x) \neq \lim_{x \to -2^+} f(x)$$

The function is discontinuous at $x = 2$ because the left-hand and right-hand limits are not equal. It is continuous from the right, though.

$$\lim_{x \to 2^-} f(x) \neq \lim_{x \to 2^+} f(x) = f(2)$$

The function is discontinuous at $x = 4$ because the left-hand and right-hand limits are not equal. It is continuous from the right, though.

$$\lim_{x \to 4^-} f(x) \neq \lim_{x \to 4^+} f(x) = f(4)$$